Three-point correlation functions in
$N = 1$ Super Liouville Theory

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Abstract

In this letter we propose exact three-point correlation functions for $N = 1$ supersymmetric Liouville theory. Along the lines of [12] we propose a generalized special function which describe the three-point amplitudes. We consider briefly the so called reflection amplitudes in the supersymmetric case.

1 Introduction

One of the main problems in string theory is how to reduce the theory to a realistic $D = 4$. In his remarcable paper Polyakov [1] showed that the answer lies in solving the 2D quantum Liouville field theory. Many attempts to solve this problem have been made in the course of the years, but some significant success in the subject up to now seems to be absent. There are several approaches to 2D quantum Liouville theory, namely continuum formulation (path integral and operator approaches)[2, 3, 4, 5], matrix models [6] and topological field theory [7]. The interest have been oriented also to conformal matter coupled to Liouville theory [8]. It is well known that assuming free field operator product expansion for the Liouville field the scaling dimensions found are in complete agreement with the results from the different approaches. Nevertheless, the Liouville dynamics is not that of a free field.

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Recently two important results in the bosonic Liouville theory have been obtained. The first one consists in expressing the correlators of matter theory coupled to gravity as free field ones [9, 10]. For this purpose one can expand the correlation function in powers of the cosmological constant. The integration over the constant Liouville mode leads to interpretation of the powers of the Liouville exponential interaction as a screening charge. This allows to use Dotsenko-Fateev thechnique in Coulomb gas picture. The three-point function has been computed in different approaches and it has been found to be in agreement with the results from matrix models [6] and [9, 10].

The second remarcable result is the exact three-point function of Liouville vertex operators which has been proposed in [11] and independently in [12]. This result represents an important basis for further investigation of the conformal blocks and their factorization properties.

In this letter we are extending the results of [11, 12] to the case of $N = 1$ supergravity using the super Liouville approach.

One advantage of this approach is that whatever thechnique is used in the bosonic case, at least conceptually, can be generalized to the supersymmetric case. Such computations as far as we know have no analogue in the matrix model formulation of 2D supergravity or superconformal matter coupled to super Liouville theory. Moreover the supersymmetric theory has richer field content than the bosonic one.

This paper is organized as follows. In Section 1 some basic facts and notations from super Liouville theory are introduced. In Sections 2 and 3 we propose exact expressions for the three-point correlation functions in Neveu-Schwarz and Ramond sectors of the theory. We have generalized the Zamolodchikov's Upsilon function to our case and some properties of our $\mathcal{R}(x, a)$ function are discussed. For some special values of the parameter a $\mathcal{R}(x, a)$ coincide with $\Upsilon(x)$.

The pole structure of the three-point function is considered. Using the reflection properties of the Liouville vertex operators $V_\alpha = V_{Q-\alpha}; R_\alpha = R_{Q-\alpha}$ we introduce the so called reflection amplitudes in Neveu-Schwarz and Ramond sectors.

2 Super Liouville theory

This section is a review of some basic results about super Liouville field theory. We shall formulate the supersymmetric theory by the action:

$$S_{SL} = \frac{1}{4\pi} \int \hat{E} \left[\frac{1}{2} D_\alpha \Phi D^\alpha \Phi - Q \hat{R} \Phi + \mu e^{b\Phi} \right] \tag{1}$$

where the real superfield Φ possess the expansion:

$$\Phi = \phi + \theta\psi + \bar{\theta}\bar{\psi} + \theta\bar{\theta} F$$

Following [2] we have chosen a background zweibein $\hat{E}$ ($\hat{R}$ is the scalar curvature corresponding to the background metric) and μ in (1) is the cosmological constant. The classical equations of motion for (1) are[3]:

$$D_\alpha D^\alpha \Phi = Q\hat{R} + \mu e^{b\Phi} \tag{2}$$

[3]One can choose the background metric to be flat and therefore $\hat{R}$ will not be essential in the sequal.

The superspace notations that we shall use are:

$$Z = (z, \theta) \tag{3}$$

$$Z_1 - Z_2 = z_1 - z_2 - \theta_1\theta_2 \tag{4}$$

The super energy-momentum tensor of the super Liouville theory is expressed in terms of the real superfield Φ:

$$T_{SL} = -\frac{1}{2}D\Phi\partial\Phi + \frac{Q}{2}D\partial\Phi \tag{5}$$

the central charge of the super-Virasoro algebra being given by:

$$\hat{c} = 1 + 2Q^2 \tag{6}$$

The superconformal primary fields are divided in two sector depending on the boundary conditions of the supercurrent. In the Neveu-Schwarz sector they are represented by the vertex operators:

$$V_\alpha = e^{\alpha\Phi(Z,\bar{Z})} \tag{7}$$

of dimension $\Delta_\alpha = \frac{1}{2}\alpha(Q-\alpha)$ and in the Ramond sector by:

$$R_\alpha^\epsilon = \sigma^\epsilon e^{\alpha\Phi(z,\bar{z})} \tag{8}$$

where σ^ϵ is so called spin field and $\Delta_\alpha = \frac{1}{2}\alpha(Q-\alpha) + \frac{1}{16}$ $(\epsilon = \pm)$. The requirement for the cosmological term in (1) to be (1/2,1/2) form in order to be able to integrate over the surface, gives a connection between Q and b:

$$Q = b + \frac{1}{b} \tag{9}$$

It is easy to see that the operators $V_\alpha = e^{\alpha\Phi}$ and $V_{Q-\alpha} = e^{(Q-\alpha)\Phi}$ have equal dimensions and therefore they are reflection image of each other (the same is true also for R_α^ϵ and $R_{Q-\alpha}^\epsilon$).

As in the bosonic case we can impose the fixed area condition (which in this case is of dimension of a lenght rather than of an area). Following the considerations in [8] we can impose the above condition inserting a δ-function into the path integral. The integration over the constant mode ϕ_0 from 0 to ∞ (the Liouville superfield Φ decomposes as follows: $\Phi(Z) = \phi_0 + \Phi'(Z)$) gives:

$$\langle\prod_{i=1}^{N} e^{\alpha_i\Phi(Z_i)}\rangle = \left(\frac{\mu}{2\pi}\right)^s \frac{\Gamma(-s)}{b}\langle\left(\int \hat{E}e^{b\Phi'}\right)^s \prod_{i=1}^{N} e^{\alpha_i\Phi'(Z_i)}\rangle_{S'_{SL}} \tag{10}$$

where:

$$S'_{SL} = \frac{1}{4\pi}\int \hat{E}\left[\frac{1}{2}D_\alpha\Phi' D^\alpha\Phi' - Q\hat{R}\Phi'\right] \tag{11}$$

$$sb = Q - \sum_{i=1}^{N}\alpha_i \tag{12}$$

which reveals the μ-dependence of the correlation function. We point out that the correlation function on the rhs in (10) is with respect to the free superfield action. Following the same strategy as in the bosonic case one has to evaluate the above amplitude as for free field but s is supposed to be positive integer and then perform an analytical continuation with respect to s. In this picture the cosmological term appears as a screening charge. Fortunately, for $N = 1$ supersymmetric conformal theories the super Coulomb gas formalism and supersymmetric Dotsenko-Fateev integrals are well developed [13].

3 The exact three-point function in the Neveu-Schwarz sector

Here we shall follow the approach of [10]which is a slightly modified version of the one described above. Consider first the three-point correlation function of Liouville vertex operators from Neveu-Schwarz sector. The perturbative expansion in the cosmological constant μ is given by:

$$\begin{aligned}
&\langle V_{\alpha_1}(Z_1)V_{\alpha_2}(Z_2)V_{\alpha_3}(Z_3)\rangle \\
&\qquad = \int D_{\hat{E}}\Phi e^{-S_{SL}} e^{\alpha_1\Phi(Z_1)} e^{\alpha_2\Phi(Z_2)} e^{\alpha_3\Phi(Z_3)} \\
&\qquad = \sum_{s=0}^{\infty} \left(\frac{\mu}{2\pi}\right)^s \frac{1}{s!} \int D_{\hat{E}}\Phi e^{-S'_{SL}} \left(\int \hat{E} e^{b\Phi}\right)^s \prod_{i=1}^{3} e^{\alpha_i\Phi(Z_i)}
\end{aligned} \tag{13}$$

where the free superfield action S'_{SL} is as in (11). Specializing to the case of correlation functions on the sphere we shall concenrate the curvature at infinity (∞,0) and the considerations will be done for flat zweibein on the plane. Therefore, we can use the super Coulomb gas formalism in order to evaluate the correlation function on the rhs in (13). As it is well known, (13) is nonzero only if:

$$sb = Q - \sum_{i=1}^{3} \alpha_i \tag{14}$$

for any order s of the pertubation series (13). The result for the s^{th} term in the expansion (13) is [11,12]:

$$\begin{aligned}
&\langle V_{\alpha_1}(Z_1)V_{\alpha_2}(Z_2)V_{\alpha_3}(Z_3)\rangle_s \\
&\qquad = \left(\frac{\mu}{2\pi}\right)^s \frac{1}{s!} \prod_{i<j}^{3} |Z_i - Z_j|^{-2\alpha_i\alpha_j} \\
&\qquad\quad \times \int \prod_{j=1}^{s} D^2 Y_j \prod_{i=1}^{3} |Z_i - Y_j|^{-2b\alpha_i} \prod_{i<j}^{s} |Y_i - Y_j|^{-2b^2}
\end{aligned} \tag{15}$$

For $N = 1$ case there exists a supersymmetric extension of the Dotsenko-Fateev integrals [13] and an analogous integral expression for the structure constants can be extracted.

Applied to our problem this integral expression gives for the three-point function in the case of integer number of screening charges the following result [14]:

$$\begin{aligned} &\langle V_{\alpha_1}(Z_1)V_{\alpha_2}(Z_2)V_{\alpha_3}(Z_3)\rangle_s \\ &= \left(\frac{\mu}{8}\Delta\left(\frac{b^2}{2}+\frac{1}{2}\right)\right)^s \prod_{i<j}^{3} |Z_i - Z_j|^{-2\delta_{ij}} \prod_{j=1}^{s} \Delta\left(\frac{j}{2} - \left[\frac{j}{2}\right] - j\frac{b^2}{2}\right) \\ &\times \prod_{j=0}^{s-1}\prod_{i=1}^{3} \Delta\left(1 - \frac{j}{2} + \left[\frac{j}{2}\right] - b\alpha_i - j\frac{b^2}{2}\right) \times \begin{cases} 1 & s \in 2\mathbb{N} \\ \frac{\eta\bar{\eta}}{b^2} & s \in 2\mathbb{N}+1 \end{cases} \end{aligned} \tag{16}$$

where:

$$\begin{aligned} &\Delta(x) = \frac{\Gamma(x)}{\Gamma(1-x)}; \\ &\delta_{ij} = \Delta_i + \Delta_j - \Delta_k; \quad i \neq j \neq k \\ &\Delta_i = \alpha_i(Q - \alpha_i) \\ &\eta = \frac{\theta_1(Z_2 - Z_3) + \theta_2(Z_3 - Z_1) + \theta_3(Z_1 - Z_2) + \theta_1\theta_2\theta_3}{[(Z_1 - Z_2)(Z_2 - Z_3)(Z_3 - Z_1)]^{\frac{1}{2}}} \end{aligned} \tag{17}$$

In the above formula we have denoted by η the $SL(2|1)$ odd invariant for any given three points (Z_1, Z_2, Z_3). In contrast to the bosonic case here the correlation function is different for $s \in 2\mathbb{N}$ and $s \in 2\mathbb{N}+1$.

At this point we use the interpretation of the (14) along the lines of [8, 15]. It was suggested to consider (14) as a kind of "on-mass-shell" condition for the exact correlation function. This condition is means that the exact correlation function sould satisfy the condition:

$$\underset{\sum_{i=1}^{3}\alpha_i = Q - sb}{res} \langle V_{\alpha_1}V_{\alpha_2}V_{\alpha_3}\rangle = \frac{(-\mu)^s}{s!}\langle V_{\alpha_1}V_{\alpha_2}V_{\alpha_3} \underbrace{\int \hat{E}V_b \ldots \int \hat{E}V_b}_{s} \rangle_{\sum_{i=1}^{3}\alpha_i = Q - sb} \tag{18}$$

when (14) holds for $s = 0, 1, 2 \ldots$. In general (18) alone seems to be unsufficient to determine N-point function, but for three Liouville vertex operators the situation is simple: the coordinate dependence on the left hand side and right hand side is as in the three-point function (13). Therefore we have the following "on-mass-shell" condition for the structure constants:

$$\underset{sb = Q - \sum_i \alpha_i}{res} C^{even(odd)}(\alpha_1, \alpha_2, \alpha_3) = I_s^{even(odd)}(\alpha_1, \alpha_2, \alpha_3) \tag{19}$$

where we have denoted by $I_s^{even(odd)}(\alpha_1, \alpha_2, \alpha_3)$ the coordinate independent part of the s^{th} term in the expansion (16).

Now we have to generalize the special function $\Upsilon(x)$ introduced in [12]. For both,

bosonic and supersymmetric cases, we define the function ($0 < Re(x) < Q$):

$$log\mathcal{R}(x,a) = \frac{1}{2}\int\limits_0^\infty \frac{dt}{t}\left\{\left[\left(\frac{Q}{2}-x\right)^2+\left(\frac{Q}{2}-a\right)^2\right]e^{-t}\right. \tag{20}$$

$$\left. -2\frac{sh^2\left[\left(\frac{Q}{2}-x\right)+\left(\frac{Q}{2}-a\right)\right]\frac{t}{4}+sh^2\left[\left(\frac{Q}{2}-x\right)-\left(\frac{Q}{2}-a\right)\right]\frac{t}{4}}{sh\frac{t}{2b}sh\frac{bt}{2}}\right\}$$

The simplest properties that are clear from (20) are:

$$\begin{aligned} &\mathcal{R}(\frac{Q}{2},\frac{Q}{2}) = 1 \\ &\mathcal{R}(x,a) = \mathcal{R}(Q-x,a) \\ &\mathcal{R}(x,a) = \mathcal{R}(a,x) \end{aligned} \tag{21}$$

We define also:

$$\mathcal{R}_0 = \left.\frac{d\mathcal{R}(x,a)}{dx}\right|_{x=a=0}$$

We propose the following expression as an exact three-point function in supersymmetric Liouville theory:

$$C^{even}(\alpha_1,\alpha_2,\alpha_3) = \left[\frac{\mu}{8}\Delta\left(\frac{b^2}{2}+\frac{1}{2}\right)b^{-1-b^2}\right]^{\frac{Q-\sum_i\alpha_i}{b}} \tag{22}$$

$$\times\frac{\mathcal{R}_0\mathcal{R}(2\alpha_1,0)\mathcal{R}(2\alpha_2,0)\mathcal{R}(2\alpha_3,0)}{\mathcal{R}(\alpha_1+\alpha_2+\alpha_3-Q,0)\mathcal{R}(x_1,0)\mathcal{R}(x_2,0)\mathcal{R}(x_3,0)}$$

$$C^{odd}(\alpha_1,\alpha_2,\alpha_3) = \left[\frac{\mu}{8}\Delta\left(\frac{b^2}{2}+\frac{1}{2}\right)b^{-1-b^2}\right]^{\frac{Q-\sum_i\alpha_i}{b}} \tag{23}$$

$$\times\frac{\mathcal{R}_0\mathcal{R}(2\alpha_1,0)\mathcal{R}(2\alpha_2,0)\mathcal{R}(2\alpha_3,0)}{\mathcal{R}(\alpha_1+\alpha_2+\alpha_3-Q,b)\mathcal{R}(x_1,b)\mathcal{R}(x_2,b)\mathcal{R}(x_3,b)}$$

where:

$$x_i = \alpha_j+\alpha_k-\alpha_i; \qquad i\neq j\neq k \tag{24}$$

Thus, we have in general:

$$\langle V_{\alpha_1}V_{\alpha_2}V_{\alpha_3}\rangle = \left(C^{even}(\alpha_1,\alpha_2,\alpha_3)+\eta\bar{\eta}C^{odd}(\alpha_1,\alpha_2,\alpha_3)\right)\prod_{i<j}|Z_i-Z_j|^{\delta_{ij}} \tag{25}$$

This expression for the exact three-point function is based on the properties of the defined $\mathcal{R}(x,a)$ function described below.

We pass to the Ramond sector leaving the reflection amplitudes and the functional properties of $\mathcal{R}(x,a)$ to the Section 5.

4 Three-point function in the Ramond Sector

According to the explicit form of the Ramond vertex operator (8) its three-point function has the following perturbative expansion:

$$\langle R^{\epsilon_1}_{\alpha_1}(z_1)R^{\epsilon_2}_{\alpha_2}(z_2)V_{\alpha_3}(Z_3)\rangle = \sum_{s=0}^{\infty}\int du_1\dots du_s\langle\prod_{i=1}^{3}e^{\alpha_i\phi(z_i)}\prod_{j=1}^{s}e^{b\phi(u_j)}\rangle \tag{26}$$
$$\times\langle\sigma^{\epsilon_1}(z_1)\sigma^{\epsilon_2}(z_2)\left(1+\theta_3\bar{\theta}_3\psi(z_3)\right)\prod_{j=1}^{s}\psi(u_j)\rangle$$

As before, in order the free bosonic correlator to be nonzero we have to impose the condition (14). It can be interpreted again as a "on-mass-shell" condition for the exact correlation function. The explicit expression for the integrals in (26) can be extracted from the corresponding formulae for the Ramond fields [16]. The final result is:

$$\langle R^{\epsilon_1}_{\alpha_1}(Z_1)R^{\epsilon_2}_{\alpha_2}(Z_2)V_{\alpha_3}(Z_3)\rangle_s = \left(\frac{\mu}{8}\Delta\left(\frac{b^2}{2}+\frac{1}{2}\right)\right)^s\prod_{i<j}^{3}|Z_i-Z_j|^{-2\delta_{ij}} \tag{27}$$
$$\times\prod_{j=1}^{s}\Delta\left(\frac{j}{2}-\left[\frac{j}{2}\right]-j\frac{b^2}{2}\right)\prod_{j=0}^{s-1}\prod_{i=1}^{2}\Delta\left(1+\frac{j}{2}-\left[\frac{j}{2}\right]-b\alpha_i-j\frac{b^2}{2}-\frac{1}{2}\right)$$
$$\times\Delta\left(1-\frac{j}{2}+\left[\frac{j}{2}\right]-b\alpha_3-j\frac{b^2}{2}\right)\times A_{\epsilon_1,\epsilon_2}$$

where:

$$A_{\epsilon,\epsilon} = \begin{cases}1, & s=2\mathbb{N}\\ \theta_3\bar{\theta}_3\frac{|z_1-z_3||z_2-z_3|}{|z_1-z_2|}, & s=2\mathbb{N}+1\end{cases} \tag{28}$$

$$A_{\epsilon,-\epsilon} = \begin{cases}1, & s=2\mathbb{N}+1\\ \theta_3\bar{\theta}_3\frac{|z_1-z_3||z_2-z_3|}{|z_1-z_2|}; & s=2\mathbb{N}\end{cases} \tag{29}$$

and $\Delta(x)$, δ_{ij}, Δ_i as in (17). Finally, we propose the following expression for the exact three-point function in the Ramond sector:

$$\langle R^{\epsilon_1}_{\alpha_1}(z_1)R^{\epsilon_2}_{\alpha_2}(z_2)V_{\alpha_3}(Z_3)\rangle = \left(C^{\epsilon_1,\epsilon_2}+\theta_3\bar{\theta}_3\frac{|z_1-z_3||z_2-z_3|}{|z_1-z_2|}\tilde{C}^{\epsilon_1,\epsilon_2}\right)\prod_{i<j}|z_i-z_j|^{\delta_{ij}} \tag{30}$$

where:

$$C^{\epsilon,\epsilon}(\alpha_1,\alpha_2,\alpha_3) = \left[\frac{\mu}{8}\Delta\left(\frac{b^2}{2}+\frac{1}{2}\right)b^{-1-b^2}\right]^{\frac{Q-\sum_i\alpha_i}{b}} \tag{31}$$

$$\times\frac{\mathcal{R}_0\mathcal{R}(2\alpha_1,b)\mathcal{R}(2\alpha_2,b)\mathcal{R}(2\alpha_3,0)}{\mathcal{R}(\alpha_1+\alpha_2+\alpha_3-Q,0)\mathcal{R}(x_1,b)\mathcal{R}(x_2,b)\mathcal{R}(x_3,0)}$$

$$C^{\epsilon,-\epsilon}(\alpha_1,\alpha_2,\alpha_3) = \left[\frac{\mu}{8}\Delta\left(\frac{b^2}{2}+\frac{1}{2}\right)b^{-1-b^2}\right]^{\frac{Q-\sum_i \alpha_i}{b}} \tag{32}$$

$$\times \frac{\mathcal{R}_0\mathcal{R}(2\alpha_1,b)\mathcal{R}(2\alpha_2,b)\mathcal{R}(2\alpha_3,0)}{\mathcal{R}(\alpha_1+\alpha_2+\alpha_3-Q,b)\mathcal{R}(x_1,0)\mathcal{R}(x_2,0)\mathcal{R}(x_3,b)}$$

and $\tilde{C}^{\epsilon_1,\epsilon_2}$ can be determined using the supersymmetry (x_i is as in (24)).

5 Pole structure and reflection amplitudes

In this section we are going to discuss the pole structure of the three-point correlation function and to define the so called reflection amplitudes. For this purpose we start with some transformation properties and some functional relations for $\mathcal{R}(x,a)$ defined in section 2 (see (20)). Some of the properties are given by (21). Using the integral representation of $\mathcal{R}(x,a)$ one can check that the following functional relation holds:

$$\mathcal{R}(x+b,a) = b^{-bx+ab}\Delta\left(\frac{bx-ba+1}{2}\right)\mathcal{R}(x,a+b) \tag{33}$$

It is clear that due to the "self-duallity" of $\mathcal{R}(x,a)$ (i.e. the invariance under $b \to 1/b$) one can conclude that:

$$\mathcal{R}(x+1/b,a) = b^{\frac{x}{b}-\frac{a}{b}}\Delta\left(\frac{x-a+b}{2b}\right)\mathcal{R}(x,a+1/b) \tag{34}$$

In the expressions for the correlation functions (22,23,31,32) the following special cases of $\mathcal{R}(x,a)$ are used:

$$\mathcal{R}(x,0) = \mathcal{R}(x,Q) = \Upsilon_1(x) \tag{35}$$

$$\mathcal{R}(x,b) = \mathcal{R}(x,Q-b) = \Upsilon_2(x) \tag{36}$$

Using the above functional relations it is easy to prove that:

$$\Upsilon_1(x+b) = b^{-bx}\Delta\left(\frac{bx+1}{2}\right)\Upsilon_2(x) \tag{37}$$

$$\Upsilon_2(x+b) = b^{1-bx}\Delta\left(\frac{bx}{2}\right)\Upsilon_1(x) \tag{38}$$

and

$$\Upsilon_1(x+1/b) = b^{\frac{x}{b}}\Delta\left(\frac{x+b}{2b}\right)\Upsilon_2(x) \tag{39}$$

$$\Upsilon_2(x+1/b) = b^{\frac{x}{b}-1}\Delta\left(\frac{x}{2b}\right)\Upsilon_1(x) \tag{40}$$

We would like to point out that:

$$\mathcal{R}(x,x) = \Upsilon(x) \tag{41}$$

where $\Upsilon(x)$ is the Zamolodchikov's Upsilon function [12].

It is easy to verify that, using the above properties, $\Upsilon_1(x) = \mathcal{R}(x,0)$ and $\Upsilon_2(x) = \mathcal{R}(x,b)$ are entire functions of x with the following zero-structure:

$$\begin{aligned}\Upsilon_1(x) = 0 \quad for \quad x = -nb - \frac{m}{b}; \quad n - m = even \\ \Upsilon_2(x) = 0 \quad for \quad x = -nb - \frac{m}{b}; \quad n - m = odd\end{aligned} \tag{42}$$

and due to (21):

$$\begin{aligned}\Upsilon_1(x) = 0 \quad for \quad x = (n+1)b + \frac{m+1}{b}; \quad n - m = even \\ \Upsilon_2(x) = 0 \quad for \quad x = (n+1)b + \frac{m+1}{b}; \quad n - m = odd\end{aligned} \tag{43}$$

(n, m are non-negative integers).

Using all the above properties of $\mathcal{R}(x,a)$ it is straightforward to check that the proposed exact three-point functions satisfy the "on-mass-shell" condition (14).

As in the bosonic case [12], the proposed correlators as a function of $\alpha = \sum_i^3 \alpha_i$ have more poles than expected: at $\alpha = Q - n/b - mb$ and at $\alpha = 2Q + n/b + mb$. They appear when more general multiple integrals are considered:

$$\underset{\sum_i \alpha_i = Q - \frac{n}{b} - mb}{res} \langle V_{\alpha_1} V_{\alpha_2} V_{\alpha_3} \rangle = \frac{(\tilde{\mu})^n (\mu)^m}{n!m!} \langle \prod_{i=1}^{3} V_{\alpha_i}(Z_i) \prod_{k=1}^{n} \int V_{1/b}(X_k) \prod_{l=1}^{m} \int V_b(Y_l) \rangle \tag{44}$$

where:

$$\frac{\tilde{\mu}}{8} \Delta\left(\frac{1}{2b^2} + \frac{1}{2}\right) = \left(\frac{\mu}{8} \Delta\left(\frac{b^2}{2} + \frac{1}{2}\right)\right)^{\frac{1}{b^2}} \tag{45}$$

We note that the correlation function (44) is self-dual with respect to $b \to \frac{1}{b}, \mu \to \tilde{\mu}$. As it was mentioned in Section 1 the Liouville vertex operators V_α and $V_{Q-\alpha}$ are reflection image of each other. We shall use this property in order to define the so called reflection amplitudes in the supersymmetric case.

We start with the reflection amplitude in the Neveu-Schwarz sector. Due to the reflection properties we define:

$$C^{even(odd)}(Q - \alpha_1, \alpha_2, \alpha_3) = S^{NS}(\alpha_1) C^{even(odd)}(\alpha_1, \alpha_2, \alpha_3) \tag{46}$$

where $S^{NS}(\alpha_1)$ is the reflection amplitude. In more details, using (22) we have:

$$C^{even}(Q-\alpha_1,\alpha_2,\alpha_3)=\left[\frac{\mu}{8}\Delta\left(\frac{b^2}{2}+\frac{1}{2}\right)b^{-1-b^2}\right]^{\frac{Q-\sum_i\alpha_i}{b}}\left[\frac{\mu}{8}\Delta\left(\frac{b^2}{2}+\frac{1}{2}\right)\right]^{\frac{2\alpha_1-Q}{b}} \tag{47}$$

$$\times\, b^{(Q-2\alpha_1)Q}\frac{\mathcal{R}_0\mathcal{R}(2Q-2\alpha_1,0)}{\mathcal{R}(Q\underbrace{-\alpha_1+\alpha_2+\alpha_3}_{x_1}-Q,0)\mathcal{R}(\alpha_2+\alpha_3+\alpha_1-Q,0)}$$

$$\times\frac{\mathcal{R}(2\alpha_2,0)\mathcal{R}(2\alpha_3,0)}{\mathcal{R}(Q\underbrace{-\alpha_1+\alpha_3-\alpha_2}_{-x_3},0)\mathcal{R}(Q\underbrace{-\alpha_1+\alpha_2-\alpha_3}_{-x_2},0)x_3,0)}$$

$$=\left[\frac{\mu}{8}\Delta\left(\frac{b^2}{2}+\frac{1}{2}\right)b^{-1-b^2}\right]^{\frac{Q-\sum_i\alpha_i}{b}}b^{(Q-2\alpha_1)Q}\left[\frac{\mu}{8}\Delta\left(\frac{b^2}{2}+\frac{1}{2}\right)\right]^{\frac{2\alpha_1-Q}{b}}$$

$$\times\frac{\mathcal{R}_0\mathcal{R}(2\alpha_2,0)\mathcal{R}(2\alpha_3,0)\mathcal{R}(Q-(2\alpha_1,0-Q))}{\mathcal{R}(x_1,0)\mathcal{R}(\alpha_2+\alpha_3+\alpha_1-Q,0)\mathcal{R}(Q-x_3,0)\mathcal{R}(Q-x_2,0)}$$

Due to (21)

$$C^{even}(Q-\alpha_1,\alpha_2,\alpha_3)=\left[\frac{\mu}{8}\Delta\left(\frac{b^2}{2}+\frac{1}{2}\right)b^{-1-b^2}\right]^{\frac{Q-\sum_i\alpha_i}{b}}\left[\frac{\mu}{8}\Delta\left(\frac{b^2}{2}+\frac{1}{2}\right)\right]^{\frac{2\alpha_1-Q}{b}} \tag{48}$$

$$\times\, b^{(Q-2\alpha_1)Q}\frac{\mathcal{R}_0\mathcal{R}(2\alpha_2,0)\mathcal{R}(2\alpha_3,0)\mathcal{R}(2\alpha_1-Q,0)}{\mathcal{R}(\alpha_1+\alpha_2+\alpha_3-Q,0)\mathcal{R}(x_1,0)\mathcal{R}(x_2,0)\mathcal{R}(x_3,0)}$$

Therefore the reflection amplitude is deduced form the above as:

$$b^{(Q-2\alpha_1)Q}\left[\frac{\mu}{8}\Delta\left(\frac{b^2}{2}+\frac{1}{2}\right)\right]^{\frac{2\alpha_1-Q}{b}}\mathcal{R}(2\alpha_1-Q)=S^{NS}(\alpha_1)\mathcal{R}(2\alpha_1) \tag{49}$$

Repeating the same considerations for $C^{odd}(\alpha_1,\alpha_2,\alpha_3)$ we found the same expression. Using the functional relations of $\mathcal{R}(x,a)$ we find that the reflection amplitude equals to:

$$S^{NS}(\alpha)=\left[\frac{\mu}{8}\Delta\left(\frac{b^2}{2}+\frac{1}{2}\right)\right]^{\frac{2\alpha_1-Q}{b}}b^{2+\frac{2(Q-2\alpha)}{b}}\frac{\Delta\left(b\alpha-\frac{b^2}{2}+\frac{1}{2}\right)}{\Delta\left(2-\frac{\alpha}{b}+\frac{1}{2b^2}-\frac{1}{2}\right)} \tag{50}$$

As in the bosonic case we associate the reflection amplitude with the two-point correlation function [11, 12].

Now we pass to the Ramond sector and consider the correlation functions (31,32) together. In this functions we have two Ramond fields ($R^{\epsilon_1}_{\alpha_1}, R^{\epsilon_2}_{\alpha_2}$) and one Neveu-Schwarz field (V_{α_3}). Therefore the reflection of the first two fields will give us one reflection amplitude, but the reflection of the third field will differs from the first one. For instance:

$$C^{\epsilon,\pm\epsilon}(Q-\alpha_1,\alpha_2,\alpha_3)=S^R(\alpha_1)C^{\epsilon,\mp\epsilon}(\alpha_1,\alpha_2,\alpha_3) \tag{51}$$

where:

$$S^R(\alpha_1) = \left[\frac{\mu}{8}\Delta\left(\frac{b^2}{2}+\frac{1}{2}\right)\right]^{\frac{2\alpha_1-Q}{b}} b^{\frac{2(Q-2\alpha)}{b}} \frac{\Delta\left(b\alpha_1 - \frac{b^2}{2}\right)}{\Delta\left(1-\frac{\alpha_1}{b}+\frac{1}{2b^2}\right)} \tag{52}$$

For the reflection of the Neveu-Schwarz field we found:

$$C^{\epsilon,\pm\epsilon}(\alpha_1,\alpha_2,Q-\alpha_3) = S^{NS}(\alpha_3)C^{\epsilon\pm\epsilon}(\alpha_1,\alpha_2,\alpha_3) \tag{53}$$

where $S^{NS}(\alpha)$ is as in (50).

Once again we would like to mention that, as in the bosonic case, the reflection amplitudes are associated with the two-point correlation function. The two reflection amplitudes found in (52,53) correspond to the correlator of two Ramond fields and of two Neveu-Schwarz fields respectively. This can be seen if we consider the correlation function of two fields devided by the volume of the group leaving invariant two marked points. Using the mechanism of canceled integration in the path integral formulation , as it has been done in [11], we can find an expression in terms of $\mathcal{R}(x,a)$. After some simple but tedious transormations one can arrive to the expressions (50,52). We leave the detailed calculations to a forthcomming paper [17].

Conclusion

We proposed an expression for the exact three-point correlation functions (22,23,31,32) in the case of super Liouville theory. The corresponding correlators of the Neveu-Schwarz and Ramond sectors are considered and some basic properties are discussed. In analogy with the bosonic case the corresponding reflection amplitudes are introduced. Taking into account that the proposed correlators (22,23,31,32) are only conjectures, some analysis on the subject and additional check is needed. We address this discussion as well as some detailed calculations and open questions to a forthcomming paper [17].

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